

# Instanton-induced Azimuthal Spin Asymmetry in Deep Inelastic Scattering

Dmitry Ostrovsky and Edward Shuryak  
*Department of Physics and Astronomy,*  
*State University of New York at Stony Brook,*  
*New York 11794, USA*  
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It is by now well understood that spin asymmetry in deep inelastic scattering (DIS) can appear if two things are both present: (i) a chirality flip of the struck quark; (ii) a nonzero T-odd phase due to its final state interaction. So far (i) was attributed to a new structure/wave function of the nucleon and (ii) to some gluon exchanges. We propose a new mechanism utilizing strong vacuum fluctuations of the gluon field described semiclassically by instantons, and show that both (i) and (ii) are present. The magnitude of the effect is estimated using known parameters of the instanton ensemble in the QCD vacuum and known structure and fragmentation functions, without any new free parameters. The result agrees in sign and (roughly) in magnitude with the available data on single particle inclusive DIS. Furthermore, our predictions uniquely relate effects for longitudinally and transversely polarized targets.

## I. INTRODUCTION

Perturbative QCD is well known to account correctly for the dependence of structure and fragmentation functions on the hard scale  $Q^2$ . On the other hand, the conventional perturbative cascade of gluon emission and of the quark pairs production (on which it is based) is clearly inadequate to explain the original quark sea and glue itself, at a border scale to nonperturbative regime  $\mu \sim 1 \text{ GeV}$ . A lot of puzzles about the nucleon structure are revealed by experiments, and we understand neither their dynamical origin nor their magnitude for other hadrons.

Already at the inclusive level of leptonic deep inelastic scattering (DIS), we learned from the spin-independent structure functions that the quark sea is rather strongly flavor polarized. The spin-dependent DIS have further shown that the sea quarks are also strongly polarized in spin, in the direction *opposite* to the polarization of the nucleon (and the valence quarks): thus the “spin crisis”.

Significant experimental efforts are now being made, at HERA as well as at CERN (COMPASS) and Brookhaven National Laboratory (STAR and PHENIX at RHIC) to see if two asymmetries are related and also to measure the polarization of the glue.

Clearly one also needs a matching theoretical efforts to get a dynamical explanation to all these phenomena, as the pQCD cascade (which is flavor and (approximately) chirality-blind) obviously cannot provide.

One particular direction of such studies are related with the non-perturbative phenomena in the QCD vacuum described semiclassically by *instantons*. There are several qualitative arguments why instantons may be clue to a potential explanation of these puzzles. Forte and Shuryak [1] had argued that instantons provide a 100 per cent effective mechanism of a polarization transfer from quarks to gluons. Kochelev [2] noticed that a sea produced via 't Hooft vertex from say left-handed  $u_L$  valence quark is 100 per cent flavor polarized, as it can only have  $\bar{d}_L d_R$ ,  $\bar{s}_L s_R$  pairs, and also should have the op-

posite chirality. Unfortunately, none of these ideas so far resulted in some quantitative predictions, see e.g. a recent work by T.Shchafer and Zetocha [3] on the spin crisis.

One more, although indirect, argument came from lattice calculations. Negele et al (the MIT group) [4] have noticed that moments of the various structure functions change very little when the true “quantum” lattice configurations are substituted by “semiclassical” (or “cooled”) ones. This procedure, which eliminates pQCD gluons and most of quantum fluctuations from vacuum configurations, is known to preserve mostly the instantons, reasulting basically in configurations of the “instanton liquid”. If true, this observation suggests that instantons alone would be sufficient to derived all structure functions, including the spin-dependent ones.

The Single Spin Asymmetries (SSA) are large spin-dependent effects which are under intense experimental study. So far their theoretical discussion (see e.g.[10]) have aimed mostly at their proper parameterization rather than explanation. One important step was an introduction of the nontrivial T-odd structure in the initial state via appropriate structure function is called the Sivers effect [11, 12], while a similar effect in fragmentation function is called the Collins effect [13]. The corresponding function was introduced in [14]. In both cases the hard block remains the usual lowest-order pQCD scattering. One more logical alternative is the *next twist* hard collision, in which at the moment of hard scattering there is extra gluonic field (or a gluon), which can be also correlated with the spin [21].

At a deeper level, two issues have been singled out, to be included as the key ingredients of any theoretical explanation of the asymmetry, as they are both the necessary prerequisites to the very existence of SSA. Those are (i) the *chirality flip*; and (ii) the *final state interaction* of the outgoing quark. These issues are best explained if the state of the transversely polarized nucleon is viewed as a superposition of plus and minus chirality states. SSA can only result from the interference of the

two amplitudes, while the usual pQCD handbag diagram conserves chirality of the quark.

The first issue can be satisfied e.g. by the introduction of a new component of the nucleon wave function, in which the valence quark rotates orbitally and thus has a spin opposite to that of the nucleon. The second is related to the decade long theoretical stalemate over Sivers effect. Namely, Collins [13] have argued that it should be zero based on T-invariance. The proof was retracted later, and the loophole is precisely the P-exponents of the outgoing quarks, or their possible final state interaction.

Brodsky, Hwang, and Schmidt (BHS) [15, 16], and Collins [17] have then shown how the Sivers effect could be incorporated into the QCD framework. For early model of a T-odd *distribution* function see[18], as well as a bag-model calculation by Yuan[19], and a model with spin-0 AND spin-1 diquarks in[20].

BHS used a very simplified model, in which a nucleon is made of a valence quark, which carry all the angular momentum, plus the spin-zero diquark. The issue (i) is included via new p-wave wave function, and (ii) via the lowest order gluon exchange between the outgoing quark and the rest of the system (the diquark). Note that in BHS approach there is no connection between (i) and (ii): just the final state interaction is necessary to make the nontrivial sector of the nucleon wave function visible.

The philosophy of our approach came out of reflections about this very point. We thought it is quite likely that the underlying dynamics of the quark chirality flip is related with the nonperturbative interaction producing chiral symmetry breaking. (By the way, throughout this paper we will ignore nonzero quark masses and thus treat chiral symmetry as exact.) in the QCD vacuum. There are convincing arguments that this phenomenon is generated by small-size instantons, see [22] for a review. And as instantons can provide the chirality flip, they also are capable to generate large ( $O(1)$  rather than  $O(\alpha_s)$ ) phase of the P-exponent, the final state interaction of the struck quark. As both are necessary for the asymmetry in question, it makes instanton mechanism twice more attractive candidate for its explanation.

In short, the physics of our proposal is as follows: the asymmetry appears when the quark-lepton collision point happens to be close to a preexisting topological vacuum fluctuation, due to tunneling through the topological barrier of QCD. In this sense the phenomenon is generic, not related to the nucleon itself, but rather proportional to density of instantons (tunneling events) in the QCD vacuum.

As the initial quark is moving in a strong color field and can “disappear into a Dirac sea” while instead another quark, with the *opposite* chirality appears close by. This is accounted by the so called 't Hooft zero mode term in a quark propagator. Also the final state interaction effect is directly given by a quark propagator on top of the instanton background: in fact it is much enhanced compared to BHS paper because the gluon field originating from instanton is large  $A \sim 1/g$  compared to

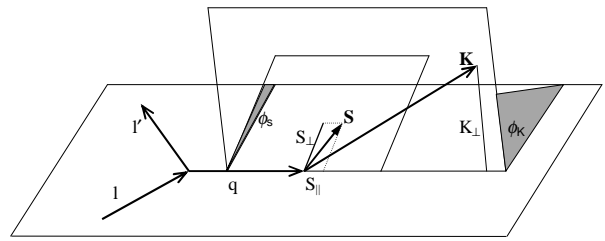


FIG. 1: Kinematics of single particle inclusive DIS in nucleon rest frame, defining all momenta and angles to be used.

perturbative field of the exchanged gluon. As a result there is no extra small factor  $\sim \alpha_s$  in the amplitude.

We will thus obtain the resulting asymmetry from well-known propagators in a relatively simple form, using then the vacuum properties from the instanton-liquid model [22] for quantitative estimates. Let us emphasise that we does not parameterize the effect via introduction of some unknown parameters or structure/fragmentation functions, but express it in terms of relatively well known parameters of the vacuum, tested in particular in other applications of instanton dynamics and/or multiple lattice works.

HERMES experiment have reported first data on SSA in SIDIS for longitudinally [5, 6, 7] and transversely polarized targets [8]. There is also data from COMPASS [9] for longitudinally polarized targets. Contributions of various dependence on the spin direction/transverse momentum of produced particle directions is possible to disentangle.

The plan of the paper is as follows: in Section 2 kinematics of SSA in SIDIS is overviewed, in Section 3 the asymmetric part of the cross section due to instantons is calculated, in Section 4 an estimate of the effect and a comparison with experiment are given.

## II. KINEMATICS OF AZIMUTHAL SSA IN SIDIS

Total cross section for deep inelastic scattering has the form

$$\frac{d\sigma}{dx dy d\phi} = \frac{\alpha_{em}^2}{Q^4} y L^{\mu\nu} W_{\mu\nu}, \quad (1)$$

where azimuthal angle  $\phi$  is unobservable in totally inclusive DIS.

Symmetric (spin independent) lepton tensor is given by (see Fig.1)

$$L^{\mu\nu} = 2(l^\mu l'^\nu + l'^\mu l^\nu) - 2g^{\mu\nu}(l \cdot l') \quad (2)$$

For totally inclusive cross section symmetric part of  $W_{\mu\nu}$  is given by

$$W_{\mu\nu} = \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) F_1(x, Q^2) + \left( P_\mu + \frac{1}{2x} q_\mu \right) \left( P_\nu + \frac{1}{2x} q_\nu \right) \frac{F_2(x, Q^2)}{P \cdot q} \quad (3)$$

In SIDIS one has one more vector parameter, the momentum of the produced hadron,  $K_\mu$ . This leads to the appearance of several new possible tensor structures of hadronic tensor  $W_{\mu\nu}$  and new dimensionless invariants on which "structure functions" may depend. The tensor structure of  $W_{\mu\nu}$  is of course limited by symmetry,  $W_{\mu\nu} = W_{\nu\mu}$  (we consider only unpolarized electrons for leptonic tensor), electromagnetic gauge invariance,  $q^\mu W_{\mu\nu} = 0$ , and parity invariance. We are interested also in *spin-dependent* asymmetries and therefore, nucleon spin  $S_\mu$  ( $S^2 = -1$ ) must be involved in nontrivial combination with produced hadron momentum. To limit the possible structures even more, we will consider hadronic tensor only to the first power in  $K$ , assuming that it enters  $W_{\mu\nu}$  in the combination  $K/Q$ , which is generally small.

Then, the possible tensor structures are  $(P + \frac{1}{2x}q)_{\{\mu\epsilon_\nu\}\rho\sigma\tau} q^\rho K^\sigma S^\tau$  and  $\epsilon_{\pi\rho\sigma\tau} P^\pi q^\rho K^\sigma S^\tau$ , the latter being multiplied either on  $(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})$  or on  $(P_\mu + \frac{1}{2x}q_\mu)(P_\nu + \frac{1}{2x}q_\nu)$ .

Although  $\epsilon^{\pi\rho\sigma\tau} P_\pi q_\rho K_\sigma S_\tau$  may not be excluded on general grounds, this structure leads to small contribution in the parton model. Indeed, in the parton model, the dependence of  $W_{\mu\nu}$  on  $P_\mu$  is possible only through the momentum of struck quark. In the infinite momentum frame  $p_\mu = xP_\mu$ . It is also true that  $K_\mu = zk_\mu$ , where  $k_\mu$  is the momentum of the quark after the collision, up to relatively small correction in the fragmentation process. Momentum conservation in the interaction vertex gives  $k_\mu = p_\mu + q_\mu$  up to small correction due to possibility of rescattering. Overall, it leads to the extra power of small transverse momentum for  $\epsilon^{\pi\rho\sigma\tau} P_\pi q_\rho K_\sigma S_\tau$ .

As we will see in the next section, instanton-induced contribution has the form

$$W_{\mu\nu} \sim (p + k)_{\{\mu\epsilon_\nu\}\rho\sigma\tau} q^\rho k^\sigma s^\tau \quad (4)$$

in accordance with the general analysis ( $s$  being spin of the quark).

Last but not least: these combinations of 2 momenta, one energy and one spin is T-odd. Therefore one can only find their contribution to any observable multiplied by another T-odd quantity, such as final state interaction phase.

### III. QUARK SCATTERING IN INSTANTON FIELD

Let us consider incoming quark with the momentum  $p$  and density matrix  $/p(1 + \gamma_5 /s)$ . Because we are interested only in spin-dependent part of the cross section we take only  $\not{p}\gamma_5 \not{s}$  as the quark density matrix.

For spin-dependent hadron tensor one has

$$\Delta W_{\mu\nu} = \text{Re} \left[ \text{tr}(\hat{k} M_{\{\mu}^1 \hat{p} \gamma_5 \hat{s} M_{\nu\}}^0) \right], \quad (5)$$

where the zeroth order vertex  $M_\nu^0 = \gamma_\nu$ , while the first order one  $M_\mu^1$  is a sum of amplitudes in instanton and

antiinstanton fields,  $\hat{k}$  is the density matrix of out-going quark (matrix elements here defined without projections on in- and out- going states). Two indices with curly brackets are assumed to be symmetrized.

Spin-dependent part of in-coming quark density matrix is chirally-odd, therefore, taking to account that all other parts of Eq. (5) are chirally-even,  $M_\mu^1$  must be chirally-odd. Therefore,  $M_\mu^1$  must contain propagation through zero-mode in instanton (antiinstanton) field. Calculation of  $M_\mu^1$  is most easily performed in chiral basis first. We return to the Dirac fermions in the end.

The calculation of  $M_\mu^1$  is in the complete analogy to the calculation of the instanton-induced chirally-odd contribution to the gluon structure function made by Moch, Ringwald, and Schrempp [23], where reader is referred for all technical details.

In instanton field one has for quark propagator due to zero mode (x-space, non-amputated, left-to-right flip)

$$S_0(x, y)_{\beta i}^{\alpha j} = \frac{\rho^2}{\pi^2 \lambda} \frac{x_\gamma \bar{\sigma}_{\beta\rho}^\gamma \epsilon^{\rho j} x_\delta (\sigma^\delta)^{\alpha\pi} \epsilon_{i\pi}}{(x^2 + \rho^2)^{3/2} (y^2 + \rho^2)^{3/2} |x||y|} \quad (6)$$

Here Greek indices are Weyl spinor indices,  $i$  and  $j$  are color indices,  $\sigma_\mu = (i, \vec{\sigma})$ ,  $\bar{\sigma}_\mu = (-i, \vec{\sigma})$  (Euclidean space),  $\epsilon^{01} = -\epsilon^{10} = -\epsilon_{01} = \epsilon_{10} = 1$  are projectors on zero mode chiral-color states. Zero mode propagator is normalized to  $\lambda$ , the lowest eigenvalue of the Dirac operator in instanton liquid. We return to the discussion of its value in Section 4.

Fourier transform with respect to incoming particle is given by

$$S_0(x, p)_{\beta\alpha i}^j = \frac{2\rho^2}{\lambda} \frac{1}{(x^2 + \rho^2)^{3/2}} \frac{x_\gamma \bar{\sigma}_{\beta\rho}^\gamma \epsilon^{\rho j} \epsilon_{i\alpha}}{|x|}, \quad (7)$$

where mass-shell condition ( $p^2 = 0$ ) is assumed and incoming particle propagator is amputated.

Non-zero mode propagator for *right-handed* quark, this is right-handed after the flip on zero mode, is (Fourier transformed and amputated for outgoing particle)

$$S_{nz}(k, x)_{\alpha i}^{\beta j} = -\frac{|x|}{\sqrt{x^2 + \rho^2}} e^{ik \cdot x} \delta_\alpha^\beta [\delta_i^j + \frac{\rho^2}{x^2} \frac{(\bar{\tau}_\rho \tau_\sigma)_i^j k^\rho x^\sigma}{2k \cdot x} (1 - e^{-ik \cdot x})], \quad (8)$$

where  $\tau_\mu = (i, \vec{\tau})$ ,  $\bar{\tau}_\mu = (-i, \vec{\tau})$  for color matrices.

We now can see the T-odd phase, which enters the propagator of the quark in instanton field. For the purpose of demonstration we will drop  $\exp(-ik \cdot x)$  from round brackets in Eq. (8). It serves to give to the propagator correct  $x \rightarrow 0$  limit. One can think that we are working in  $k \cdot x \gg 1$  kinematical domain. Then, Eq. (8) becomes

$$S_{nz}(k, x) \simeq -e^{ik \cdot x} \exp \left[ i \frac{\bar{\eta}^{a\mu\nu} k_\mu x_\nu \tau_a}{k \cdot x} \ln \left( \frac{x^2 + \rho^2}{x^2} \right) \right], \quad (9)$$

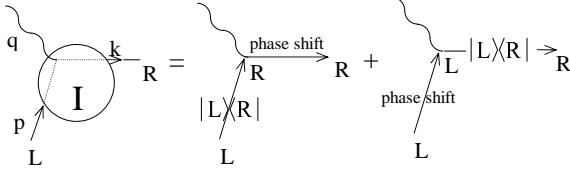


FIG. 2: The amplitude for a single quark scattering in an instanton background field (shown schematically by a circle with I) can be written as a sum of two diagrams. They differ by where the chirality flip, between a chosen left-handed (L) initial struck quark into a right-handed (R) one, which is described by the zero mode part of the propagator. The “phaseshift” subscript reminds about a complex phase of the non-flip part of the propagator, which has to be kept to get the nonzero answer.

where  $\bar{\eta}^{a\mu\nu}$  is 't Hooft symbol. It makes the phase T-odd after continuation to Minkowski space.

Contribution to  $M_\mu^1$  is given by

$$M_\mu^1 = \int d^4x e^{iq \cdot x} S_{nz}(k, x) \sigma_\mu S_0(x, p) \quad (10)$$

Before calculating the integral one has to perform some algebra. Namely, chiral-color projectors of zero modes effectively mix chiral ( $\sigma$ ) and color ( $\tau$ ) matrices. We make use of the relation

$$\epsilon_{\gamma\alpha}(\sigma_\mu)^{\gamma\dot{\gamma}}(\bar{\sigma}_\nu)_{\dot{\gamma}m} = \epsilon_{\gamma m}(\sigma_\nu)^{\gamma\dot{\gamma}}(\bar{\sigma}_\mu)_{\dot{\gamma}\alpha} \quad (11)$$

Therefore, there is no much sense of keeping difference between  $\sigma$  and  $\tau$  matrices, when required we will indicate correct indices to distinguish between color and chiral degrees of freedom.

After integration

$$M_\mu^1 = \frac{4\pi\rho^2}{\lambda} i(\sigma_\mu \bar{\sigma}_\rho \epsilon)^{\alpha i} \epsilon_{\beta j} \Phi^\rho(k, q) \quad (12)$$

with

$$\Phi^\rho(k, q) = -\frac{p^\rho}{p^2} - \frac{k^\rho}{2k \cdot q} (1 - f(\rho|q|)), \quad (13)$$

where  $f(a) = aK_1(a)$  ( $f(0) = 1$ ), and we took into account that  $k = p + q$ . First term in the expression of  $\Phi^\rho$  contains 0 in the denominator, however in the nominator one effectively has  $\bar{\sigma}_\rho p^\rho$ , which being multiplied on incoming quark state gives 0. This term does not contribute to the spin-dependent part of the cross section anyway, so we may neglect it altogether.

Eq. (12) corresponds to applying zero mode propagator to the incoming quark before its collision with a virtual photon. There is, of course, another diagram (see Fig.2), which refers to zero mode propagator inserted in the outgoing quark line. That can easily be found by Hermitian conjugation of Eq. (12) and substitution  $k \leftrightarrow -p$ , which gives

$$M_\mu^{1'} = -\frac{4\pi\rho^2}{\lambda} i\epsilon_{i\alpha}(\epsilon\sigma_\rho\bar{\sigma}_\mu)^{j\beta} \Phi^\rho(p, q) \quad (14)$$

Next, we are taking trace over color indices. Because all other parts of the diagram are trivial in color, one reduces  $M_\mu^1$  to

$$M_\mu^1 = \frac{4\pi\rho^2}{\lambda} i(\sigma_\mu \bar{\sigma}_\rho \Phi^\rho(k, q) - \sigma_\rho \bar{\sigma}_\mu \Phi^\rho(p, q)) \quad (15)$$

The matrix element for propagation in the anti-instanton field is given by  $\bar{\sigma} \leftrightarrow \sigma$ .

Inserting  $M_\mu^1$  to the Eq. (5) one has

$$\Delta W_{\mu\nu} = \frac{4\pi\rho^2}{\lambda Q^2} \text{Im}[\text{tr}(\hat{k}\hat{p}\gamma_{\{\mu}\hat{p}\gamma_5\hat{s}\gamma_{\nu\}}) + \text{tr}(\hat{k}\gamma_{\{\mu}\hat{k}\hat{p}\gamma_5\hat{s}\gamma_{\nu\}})](1 - f(\rho|q|)) \quad (16)$$

Some trivial Dirac algebra gives

$$\begin{aligned} \Delta W_{\mu\nu} &= \frac{8\pi\rho^2}{\lambda} \frac{1}{Q^2} (k+p)_{\{\mu} \text{Im tr}(\hat{k}\hat{p}\gamma_5\hat{s}\gamma_{\nu\}})(1 - f(\rho|q|)) \\ &= -\frac{32\pi\rho^2}{\lambda} \frac{1}{Q^2} (k+p)_{\{\mu} \epsilon_{\nu\}} \rho \sigma_\tau q^\rho k^\sigma s^\tau (1 - f(\rho|q|)). \end{aligned} \quad (17)$$

In the last expression we switched back to Minkowski space and made use of  $\sigma_0^E = i\sigma_0^M$ .

In this calculation we have neglected the interaction between instanton and the rest of the nucleon, apart of the struck quark. This approximation is motivated by the fact that the typical instanton size in the QCD vacuum  $\rho \approx 1/3 fm$  is small compared to nucleon size  $R_N$ . Their account would lead to corrections of the order  $\rho^2/R_N^2 \sim 1/10$ .

In vacuum parametrized by an ensemble of instantons one has to integrate Eq. (17) over collective degrees of freedom: color rotations and instanton size.

$$\left(\frac{\rho^2}{\lambda}\right) \rightarrow \frac{\kappa}{\bar{\rho}^2 m^*}, \quad (18)$$

where  $\kappa$  is the instanton diluteness factor,  $\bar{\rho}^2$  is the characteristic instanton size, and  $1/m^*$  is the inverse effective quark mass in the instanton-liquid model.

#### IV. ESTIMATE OF THE ASYMMETRY

Eq. (17) constitutes the result for spin-dependent asymmetric tensor at partonic level. To change it to hadronic result one has to substitute  $p = xP$ ,  $k + p = 2xP + q$ , and  $k = K/z$ .

To find out the correct kinematical normalization we compare analogous calculation for spin-independent DIS cross section, which gives

$$W_{\mu\nu}^{parton} = 2N_c(p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu}(p \cdot k)). \quad (19)$$

Rewriting this through conventional structure functions one finds that correct normalization is given by multiplication of  $W_{\mu\nu}^{parton}$  on

$$\sum_q e_q^2 f_q(x)/(2N_c Q^2) \quad (20)$$

and we have  $F_1(x) = F_2(x)/(2x) = \frac{1}{2} \sum_q e_q^2 f_q(x)$  for conventional structure functions in our approximation. Because of the spin dependence of Eq. (17) one has to use spin-dependent quark distributions, which we tentatively will call  $f_{q,s}(x)$ . We can now rewrite Eq. (17) as

$$\Delta W_{\mu\nu} = \frac{32\pi\kappa}{\bar{\rho}^2 m^* N_c} \frac{x}{z} \frac{1}{Q^4} (P + (1/2x)q)_{\{\mu\varepsilon\nu\}\rho\sigma\tau} q^\rho K^\sigma s^\tau \times (1 - f(\rho Q)) \sum_q e_q^2 f_{q,s}(x) D_q(z). \quad (21)$$

Asymmetric part of the cross-section is now

$$\frac{d\Delta\sigma}{dx dy dz d\phi_K} = \frac{\alpha_{em}^2}{Q^2} \frac{32\pi\kappa}{\bar{\rho}^2 m^* N_c} \frac{|K_\perp|}{zQ} (1 - f(\rho Q)) \times \sum_q e_q^2 f_{q,s}(x) D_q(z) \left( \frac{2}{Q} \frac{1-y}{y} \sin(\phi_K - \phi_s) |s_\perp| + \frac{(1-y/2)\sqrt{1-y}}{Mx} \sin\phi_K s_\parallel \right). \quad (22)$$

One can now define what exactly are the spin-dependent quark distributions  $f_{q,s}(x)$  we introduced in analogy to  $f_q(x)$ . They correspond to the probability to find a quark in a hadron polarized the same way as a hadron minus the probability to find a quark polarized in opposite direction then a hadron. Schematically,

$$f_{q,s}(x)s \rightarrow \Delta f(x)s_\parallel + \Delta_T f(x)s_\perp. \quad (23)$$

Inserting it in Eq. (22) gives

$$\frac{d\Delta\sigma}{dx dy dz d\phi_K} = \frac{\alpha_{em}^2}{Q^2} \frac{32\pi\kappa}{\bar{\rho}^2 m^* N_c} \frac{|K_\perp|}{zQ} (1 - f(\rho Q)) \times \sum_q e_q^2 D_q(z) \left( \frac{2}{Q} \frac{1-y}{y} \sin(\phi_K - \phi_s) |S_\perp| \Delta_T f_q(x) + \frac{(1-y/2)\sqrt{1-y}}{Mx} \sin\phi_K S_\parallel \Delta f_q(x) \right). \quad (24)$$

To obtain relative asymmetry one has to compare Eq. (24) to totally inclusive cross section

$$\frac{d\sigma}{dx dy dz d\phi} = \frac{\alpha_{em}^2}{Q^2} \frac{1 + (1-y)^2}{y} \sum_q e_q^2 f_q(x) D_q(z) \quad (25)$$

From Eqs. (24) and (25) one can see that for the most simplistic model of a nucleon, where total spin and charge are carried out by single quark (the other two being insulated from virtual photon in diquark state), relative asymmetry does not depend on distribution functions of the nucleon and is in a sense universal, applicable to *all other hadrons*.

More realistic approximation is  $\Delta f(x) = \Delta_T f(x)$ . It ignores differences due to relativistic motion of the quarks inside nucleons. However, in absence of reliable experimental data on  $\Delta_T f(x)$  one can use this approximation to get reasonable estimate of the transverse asymmetry. Model calculations also favor such an approximation.

From Eq. (24) one can readily see that if  $\Delta f(x) = \Delta_T f(x)$  is assumed, the relative size of transverse and longitudinal asymmetries is purely kinematical and does not depend on any details of hadronic structure.

We will now give an estimate for prefactor in Eq. (24) from the single instanton approximation (SIA) of instanton-liquid model. For general discussion of instanton phenomenology the reader can consult e.g. [22]. We will use the usual diluteness parameter and size

$$\bar{\rho} = 1/3 fm \quad \kappa = n\bar{\rho}^4 \approx 1/3^4 \quad (26)$$

As for the accuracy of SIA and the value of the (appropriately averaged) value of the Dirac eigenvalues  $m^*$ , see detailed discussion in ref.[24]. It is found there that if it would be simply a quantity with one zero mode, like  $\langle \bar{q}q \rangle$ , the accuracy of selecting one closest instanton from the ensemble and ignoring all others is typically about 30%. In this case the definition of it (called  $m_{uu}$  in [24]) should be  $m^* \equiv (\langle 1/\lambda \rangle)^{-1}$  where the angular bracket stands for real eigenvalue spectrum in the vacuum ensemble. Its numerical value changes from  $m^* = 120$  MeV for random instanton liquid model to  $m^* = 170$  MeV in interacting instanton ensemble. It must be noted that in our calculation spin asymmetry depends on both chirality flip and phase shift on the same instanton. Thus, we expect that in this case SIA is more accurate and use  $m^* = 170$  MeV. In summary, all instanton-related parameters appear in the following combination, which has the dimension of the energy

$$\frac{32\pi\kappa}{\bar{\rho}^2 m^* N_c} = 0.88 \text{ GeV} \quad (27)$$

Although it makes a parameter of the order of 1 GeV, one should keep in mind that it includes the instanton density which is nonperturbatively small  $\kappa \sim \exp(-2\pi/\alpha_s(\rho))$ . Furthermore, the phenomenological smallness (26) is not seen only because it happens to be compensated by large numerical factor  $32\pi$ .

### A. Comparison with experiment

Detailed comparison with the experiment is outside the scope of this paper. We present here only a few details to establish phenomenological relevance of our model. We consider longitudinal and transversal spin asymmetries for production of  $\pi^+$  mesons off polarized proton target [5, 6, 7]. For simplicity, we will assume that in order to produce  $\pi^+$  from the proton one has to struck a  $u$  quark. In other words,  $D_q^{\pi^+}(z) = 0$  unless  $q = u$ . Then, from Eqs. (24), (25) longitudinal asymmetry is

$$A_{UL}^{\sin\phi} = 0.88 \text{ GeV} \frac{|K_\perp|}{zQ} (1 - f(\rho Q)) \frac{y(1-y/2)\sqrt{1-y}}{M(1+(1-y)^2)} \times \frac{\Delta f_u(x)}{xf_u(x)}. \quad (28)$$

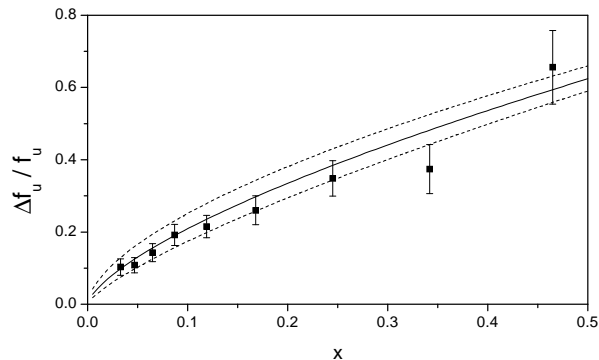


FIG. 3: Relative polarization of  $u$ -quark in the proton. The error bars combine statistical and systematic errors. Parametrization  $x^\alpha$  with  $\alpha = 0.68 \pm 0.08$  is shown by solid

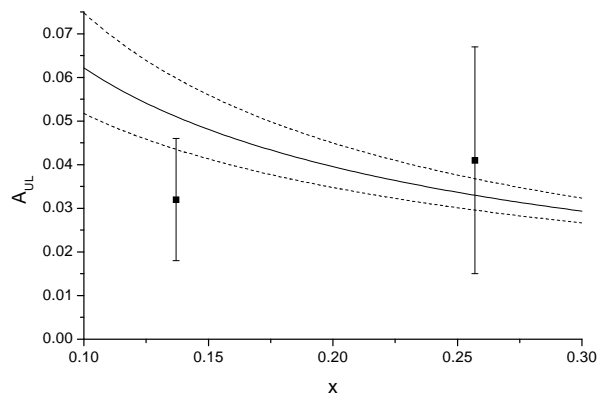


FIG. 4: Experimental values  $A_{UL}$  for moderate  $x$  are shown with comparison with the model prediction. Theoretical uncertainty is due to uncertainty in polarized distribution function.

The ratio of polarized to unpolarized distribution function is measured by HERMES collaboration [25, 26, 27] for the same kinematical region as spin asymmetries. It is shown on Fig.3. It may be fitted with reasonable accuracy by simple power law  $\Delta u/u = x^\alpha$  with  $\alpha = 0.68 \pm 0.08$ . (Note that this dependence should not be true down to very low  $x$ , or else the  $\Delta u/(xu)$  blows up.) Parameters  $Q^2$ ,  $x$ , and  $y$  are related by  $Q^2 = xy(s - M^2)$  (here  $s$  is Mandelstam variable,  $s = 2ME$  in proton rest frame).  $|K_\perp|$  and  $z$  can be taken as independent from the rest of kinematical variables as long as  $|K_\perp|/z \ll Q$ . Otherwise DIS separation of parallel and transversal degrees of freedom breaks down. In HERMES experiment  $\langle K_\perp \rangle = 0.44$  and  $\langle z \rangle = 0.48$ , while  $Q^2$  is constrained to be  $> 1\text{GeV}^2$ . Thus, we assume  $|K_\perp|/z = 0.92\text{GeV}$  in Eq. (28). Then we average over  $0.2 < y < 0.85$ . The result for moderate values of  $x$  is shown in Fig.4. We have excluded  $x < 0.1$  because for small  $x$  our simplifying assumptions about proton structure are not applicable.

Relation of transversal to longitudinal asymmetries for the same simplified model of  $\pi^+$  production we use is (see

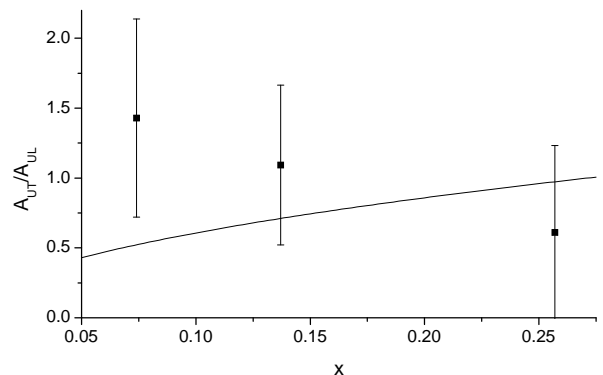


FIG. 5: Experimental values of relative transversal to longitudinal asymmetries  $A_{UT}/A_{UL}$  for moderate  $x$  are shown with comparison with the model prediction.

Eq. (24))

$$\frac{A_{UT}}{A_{UL}} = \frac{2\sqrt{1-y}\sqrt{x}}{(1-y/2)y^{3/2}} \frac{M}{\sqrt{s-M^2}} \frac{\Delta_T f_u(x)}{\Delta f_u(x)} \quad (29)$$

Recall that in our simplified model  $\Delta_T f_u(x) = \Delta f_u(x)$ . Taking to account HERMES kinematics as outlined above one finally has an estimate

$$\frac{A_{UT}}{A_{UL}} = 1.92\sqrt{x} \quad (30)$$

which is compared to the available data in Fig. 5

## V. CONCLUSIONS AND OUTLOOK

In this paper we have made a step toward the semiclassical theory of various spin-dependent effects in QCD, based on instantons.

The advantages of the instanton-induced mechanism as an explanation of azimuthal spin asymmetries are based on the fact that they provide simultaneously both required ingredients. First, instantons are the well known source of the chirality flip, relevant in the kinematic domain of the scale of about 1 GeV. Second, instantons also provide large T-odd phase for the outgoing quark. We emphasize that the use of instantons allows us not to introduce any new parameter or structure/fragmentation functions, but express the result in terms of well known quantities. The main of them is the “vacuum diluteness” parameter (26), which gets compensated by a large numerical factor  $32\pi$  in the answer.

The magnitude of the effect is thus fixed with the absolute normalization (28), based on the parameters of the instanton ensemble model known since 1982, see [22]. The result agrees in sign and magnitude with the available experimental data in suitable kinematic domain. We have argued that the asymmetry does not depend on the specific distribution functions of the nucleon, and is thus universal to all other hadrons.

Furthermore, our spin-dependent azimuthal asymmetries have a particular tensor structure in the lowest nonzero order of  $K_{\perp}/Q$  as long as parton interpretation of hadron structure is taken into account. It leads to the specific prediction for the dependence of longitudinal and transverse asymmetries on kinematical parameters which is completely independent on the phenomenological inputs.

For the outlook, one may think that the explanation of other spin asymmetries, e.g. in  $pp \uparrow$  collisions, can also be provided by instantons. The FERMILAB data [30, 31, 32] revealed considerable asymmetry in pion production starting from  $x_F \sim 0.5$  and rising towards higher values of  $x_F$ . The explanation of this asymmetry based on instanton mechanism was pioneered by Kochelev [28, 29] who provided qualitative expressions for it. More quantitative calculation would however be needed to relate this effect to spin effects in DIS we discuss above.

One more direction of future work may be combined

with the description of even the *non – polarized* DIS in the  $x_F \rightarrow 1$  limit, where it is known to be dominated by large higher twist effects. Those were also speculated long ago to be due to instantons [33], but it was never demonstrated. If that conjecture happens to be true, the instanton diluteness  $\kappa$  would drop out from numerator and denominator of the spin asymmetry, resulting in really large  $\sim O(1)$  and truly universal asymmetry independent on  $\kappa$ .

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